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PREDICTING EIGHTH-GRADE ALGEBRA ACHIEVEMENT

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The relative importance of a group of cognitive variables in explaining the algebra achievement of high-ability eighth graders was investigated. Stepwise multiple regression analyses identified algebra prognosis test scores and performance in seventh-grade mathematics as the most influential predictors of algebra grades, whereas mathematics problem solving, the prognosis test, and IQ were shown to best predict achievement on a standardized algebra test. A discriminant analysis demonstrated the importance of the prognosis test in discriminating between students who successfully completed first-year algebra and those who dropped out of or failed the course. Although the study showed no sex difference in algebra achievement test scores, girls in the sample attained higher percentage grades for the course.

In many schools, superior mathematics students are given the opportunity to study first-year algebra in the eighth grade as the first course of an accelerated mathematics program that culminates with Advanced Placement Calculus in the senior year. Unfortunately, first-year algebra proves to be a stumbling block for some of the students who enroll in the accelerated program. Unsuccessful students either drop the algebra course and revert back to regular eighth-grade mathematics or complete the course only to have to repeat it in the ninth grade. In either case, intelligent, previously high-achieving students experience the trauma of failure. An important first step in attempting to reduce the incidence of such trauma would be to gain some insights regarding the correlates of success in eighth-grade algebra.

A number of researchers have investigated the relative value of previous grades, ability measures, achievement tests, and algebra aptitude tests for predicting success in a beginning algebra course. On the one hand, the research reported in the literature has generally shown previous mathematics grades to have the highest correlation with algebra grades, whether the set of predictors included only conventional ability and achievement tests (Rothenberger, 1967) or a special-purpose aptitude test as well (Barnes & Asher, 1962; Kovaly, 1980; Mogull & Rosengarten, 1972, 1974). On the other hand, two studies that included algebra achievement test scores as a second criterion of success found algebra aptitude tests to be the best predictors of both criteria (Hanna, Bligh, Lenke, & Orleans, 1969; Sabers & Feldt, 1968). It is important to note, however, that previous grades were not among the independent variables in the latter study.

Interestingly, neither performance on mathematics achievement tests nor general intelligence surfaced as the single most important predictor of algebra achievement in any of the studies cited above. Nonetheless, total mathematics

achievement test scores (Barnes & Asher, 1962; Sabers & Feldt, 1968) and computation subtest scores (Rothenberger, 1967) were shown to correlate substantially with algebra achievement. And, Taylor, Brown, and Michael (1976) found that ability measures are better predictors of attainment in an algebra course than a host of affective and demographic variables.

Because algebra prediction studies have focused primarily on courses taught in the ninth grade, very little is known about the factors that may be related to achievement among the academically talented students who are enrolled in algebra courses in the eighth grade. Therefore, the present study was designed to investigate the relative value of seventh-grade mathematics performance, general intelligence, mathematics achievement test scores, and algebra aptitude test scores for predicting success in eighth-grade algebra. Two criteria for algebra achievement were employed: algebra grades and scores on a standardized first-year algebra test.

Although recent studies (Armstrong, 1981; Swafford, 1980) have shown no sex differences on first-year algebra achievement tests for somewhat heterogeneous samples, it has been reported that high-ability boys tend to outperform their female counterparts on tests of mathematical reasoning and problem solving (e.g., Benbow & Stanley, 1982; Fox, 1975). Thus, the relationship of sex differences to success in an accelerated algebra course was also investigated in the present study.

Finally, unlike earlier studies, scores on each of the predictor variables of boys and girls who dropped out of first-year algebra during the eighth grade were examined to gain further information about the cognitive variables that differentiate successful from unsuccessful students.

METHOD

Subjects

The sample consisted of 139 students (68 boys and 71 girls) from a suburban public school who had been selected for an accelerated eighth-grade algebra course primarily on the basis of their performance in a seventh-grade prealgebra course but also on the basis of their IQs. The subjects had a mean IQ of 127.14 (SD = 7.31); their mean age at the beginning of the eighth grade was 13.14 years (SD = 0.32). Fifteen students (7 boys and 8 girls) from the original sample dropped the algebra course during the school year because of poor achievement.

Predictor Variables

Previous mathematics grade. The students' average percentage grades in seventh-grade mathematics were used as an indication of previous mathematics performance. Although the students came from seventh-grade classes taught by two different teachers, their grades reflected test performance only.

IQ. The students' IQs, as determined by the Lorge-Thorndike Intelligence

Tests, Level C (Lorge, Thorndike, & Hagen, 1964), were obtained from school records. The test was administered in January of the seventh grade as part of the regular testing program.

Mathematics achievement test scores. Scores on the Mathematics Computation, Mathematics Concepts, and Mathematics Problem Solving portions of the Metropolitan Achievement Tests, Form G, Advanced Level (Durost, Bixler, Wrightstone, Prescott, & Balow, 1970) were obtained from school records. The test was administered in January of the seventh grade as part of the regular testing program. Standard scores furnished by the test publisher were used in the data analyses.

Prognosis test score. The Orleans-Hanna Algebra Prognosis Test (Orleans & Hanna, 1968) was administered, for the purposes of this study, during regular mathematics classes in April of the seventh grade. The test is a composite of 58 work-sample items, student-predicted mid-year algebra grades, and student-reported grades received most recently on their report card in four major subjects. The grades are scored A = 8, B = 6, C = 4, D = 2, and F = 0. Thus, the maximum total score is 98. Orleans and Hanna report test-retest reliabilities ranging from .91 to .96.

Criterion Variables

Algebra grade. The students' final percentage grades for the first-year algebra course were obtained from teachers' grade books. Although the students were taught by two different algebra teachers, both teachers used the same unit tests as their criteria for grading.

Algebra achievement test score. The Modern Algebra Test (Hanna, 1972), a standardized 45-item test designed to measure mastery of the content of a first-year algebra course, was administered, for the purposes of the study, in regular algebra classes during the last week of the school year. Total raw scores were used in the analyses. Hanna reports a split-half reliability of .87 for the test.

RESULTS

The means, standard deviations, and correlations of all variables for students who completed the eighth-grade algebra course are presented in Table 1. A consideration of the means and standard deviations on the cognitive predictors reveals the homogeneity of the select group of students in an accelerated mathematics program. First, not only was the mean IQ of the sample exceptionally high, but the standard deviation was only half that for the population at large. Second, the sample's outstanding performance on the mathematics portions of the Metropolitan Achievement Tests can be seen in mean scores that convert to national percentile ranks of 94, 98, and 93 for the computation, concepts, and problem-solving tests, respectively. Similarly, the

sample's mean score on the Orleans-Hanna Algebra Prognosis Test, when compared with a seventh-grade norming sample, is equivalent to the 92nd percentile rank. Finally, the overall achievement of the eighth graders who completed the algebra course was outstanding, as evidenced by a mean score on the Modern Algebra Test that is the equivalent of the 92nd percentile rank among students who take the course in the *ninth* grade.

Table 1
Means, Standard Deviations, and Correlations of All Variables

Variable	1	2	3	4	5	6	7	8	*M*	*SD*
1. Sex (M = 1, F = 0)									0.49	0.50
2. Math grade	.04								84.49	5.43
3. IQ	.27	−.06							127.77	7.21
4. Math computation	.09	.13	.20						113.39	8.35
5. Math concepts	.24	.12	.34	.38					116.77	7.88
6. Problem solving	.29	.19	.35	.57	.43				116.33	7.61
7. Prognosis test	−.04	.36	.23	.34	.36	.36			78.81	7.79
8. Algebra grade	−.21	.37	−.04	.29	.21	.20	.47		81.94	7.51
9. Algebra test	−.02	.14	.36	.35	.28	.42	.41	.48	33.57	4.56

Note. For $N = 124$, $r_{.05} = .18$, $r_{.01} = .23$, $r_{.001} = .29$, two-tailed.

On the one hand, an examination of the correlation matrix in Table 1 indicates that all the predictor variables except IQ were significantly correlated with the year-end algebra grade. On the other hand, the only cognitive variable not significantly related to the algebra achievement test score was performance in seventh-grade mathematics. And, although prognosis test scores provided the best indication of algebra grades, mathematics problem solving narrowly surpassed the prognosis test as the strongest predictor of algebra achievement test performance. The correlation between the two criteria of algebra achievement (i.e., grade and test score), although positive, was only moderate. Finally, the correlation matrix reveals a sex-related difference favoring boys on general intelligence, mathematics concepts, and problem solving. Nonetheless, it shows that girls attained higher algebra grades. Boys and girls did not differ significantly on any of the other variables.

The data were analyzed further by means of two stepwise regression analyses—one for each criterion of achievement. Because of the sex differences observed in the correlation matrix, sex was included in each analysis. However, it was arbitrarily entered into the prediction equations last so that the additional variance in algebra achievement accounted for by sex differences, beyond that explained by the cognitive predictors, could be assessed. The results of the regression analyses are presented in Table 2.

As indicated in Table 2, the multiple correlation of all cognitive predictors with algebra grade was .56. The prognosis test score, accounting for 22% of the variation in algebra grades, entered the equation first. Previous mathematics grade, the only other cognitive variable to contribute a statistically significant ($p < .01$) increment to the prediction equation, explained only 5%

Table 2
Stepwise Regression Analyses Predicting Year-End Algebra Grade and Algebra Achievement Test Score

Predictor	Multiple R	R^2	Change in R^2	F to enter
	Algebra grade			
1. Prognosis test	.47	.22	.22	35.38***
2. Math grade	.52	.27	.05	7.28**
3. Math computation	.54	.29	.02	3.02
4. IQ	.55	.31	.02	3.27
5. Math concepts	.56	.31	.00	<1
6. Math problem solving	.56	.31	.00	<1
7. Sex[a]	.59	.35	.04	6.97**
	Algebra achievement test score			
1. Math problem solving	.42	.17	.17	25.76***
2. Prognosis test	.50	.25	.08	12.79***
3. IQ	.54	.29	.04	6.38*
4. Math computation	.55	.30	.01	1.55
5. Math grade	.55	.30	.00	<1
6. Math concepts	.55	.30	.00	<1
7. Sex[a]	.57	.32	.02	3.64

Note. $N = 124$.
[a]Purposely entered last.
*$p < .05$. **$p < .01$. ***$p < .001$.

additional variance. The inclusion of sex in the equation significantly ($p < .01$) increased the multiple R from .56 to .59.

Reference to Table 2 further reveals that the stepwise regression of algebra achievement test scores on the same set of cognitive predictors yielded a multiple correlation coefficient of .55. This time, mathematics problem solving, accounting for 17% of the variation in algebra achievement, was the first variable to enter the prediction equation. Statistically significant contributions to the equation were also provided by the prognosis test score ($p < .001$) and IQ ($p < .05$). The inclusion of sex in the equation did not significantly improve the prediction of test performance.

Separate multiple regression analyses for male and female students were not performed because of the reduced sample sizes. However, the zero-order correlations between predictor and criterion variables for each sex separately are shown in Table 3. It can be seen that, over all, the correlation coefficients were higher for boys than girls, but the only significant difference in coefficients for the two sexes was in the correlation between mathematics concepts and algebra grade. Interestingly, for boys in the sample, general intelligence was a slightly stronger predictor of algebra achievement test performance than mathematics problem solving was.

A discriminant analysis was performed to determine the extent to which the same set of cognitive predictors could differentiate between those students who did successfully complete eighth-grade algebra and those who did not.

Table 3
Correlations of Predictors with Criterion Variables for Boys and Girls Separately

Predictor	Criterion			
	Algebra grade		Algebra test	
	Boys[a]	Girls[b]	Boys[a]	Girls[b]
Prognosis test	.58	.36	.44	.39
Math grade	.41	.35	.13	.15
Math computation	.42	.20	.40	.31
Math concepts	.44*	.06*	.34	.23
Math problem solving	.33	.21	.47	.41
IQ	.19	−.16	.49	.25

[a] $n = 61$. [b] $n = 63$.
*Differ at .05 level of significance.

Because a preliminary analysis had revealed that the dropouts (seven boys and eight girls) and the students who were required to repeat the course (five boys and two girls) did not differ significantly on any of the predictor variables, the two groups were combined for the analysis. A Wilks's lambda value of .83, $\chi^2(6) = 24.72$, $p < .001$, yielded by the discriminant analysis indicated that the six predictors discriminated significantly between the two groups of students. Further, a comparison of predicted group membership with actual group membership showed that 73% of the successful students and 77% of the unsuccessful students were correctly classified by the discriminant function equation.

The standardized discriminant function coefficients, as well as the means and standard deviations for the predictors, are presented in Table 4. It can be seen that the prognosis test was, again, the best indicator of success in the eighth-grade algebra course. Mathematics achievement test scores and IQ were less effective, albeit statistically significant, discriminators between the two groups of students. However, the successful and unsuccessful algebra students differed very little in their previous mathematics grades.

Table 4
Discriminant Analysis of Scores on Predictor Variables for Successful and Unsuccessful Algebra Students

Variable	SDFC[a]	Successful ($n = 117$)		Unsuccessful[b] ($n = 22$)		F
		M	*SD*	*M*	*SD*	
Math grade	.15	84.63	5.16	82.14	7.86	3.61
IQ	.10	127.68	7.07	124.23	8.02	4.24*
Math computation	.35	113.58	8.29	107.32	6.88	11.11**
Math concepts	.33	117.04	7.81	110.91	7.54	11.53***
Math problem solving	−.12	116.42	7.74	111.95	5.58	6.65**
Prognosis test	.61	79.16	7.67	71.32	6.10	20.55***

[a] Standardized discriminant function coefficient.
[b] Either dropped or failed the course.
*$p < .05$. **$p < .01$. ***$p < .001$.

The prognosis test scores were examined closely in an attempt to identify a reasonable cutoff point for making placement decisions. It was determined that by excluding from the accelerated program students with test scores lower than 75, 73% of the failures could have been prevented. However, the same criterion would have led to a rejection of 27% of the students who ultimately passed the course.

DISCUSSION

Of the variables included in the present study, an algebra prognosis test was identified as the best overall predictor of success in an eighth-grade algebra course. Scores on the prognosis test provided the best indication of algebra grades and were just slightly less effective than problem-solving scores in predicting achievement on a standardized algebra test. Further evidence of the predictive validity of the prognosis test was seen in its ability to discriminate between those students who successfully completed the course and those who dropped out or failed. Thus, it might be inferred that despite the additional time and expense involved with administering such a test its use for purposes of selecting students for an eighth-grade algebra course is warranted. Moreover, it provides a more objective criterion than previous grades alone when explaining selection decisions to parents. The setting of an arbitrary cutoff point poses a dilemma, however, since a criterion stringent enough to prevent most of the potential failures is also likely to exclude a number of students who would succeed.

Interestingly, two different combinations of predictors were yielded by the stepwise multiple regression analyses. More specifically, the prognosis test score and the mathematics grade for the previous year provided optimal prediction of the algebra grade, whereas the problem-solving test score, prognosis test score, and IQ, in combination, best predicted performance on the algebra achievement test. The dissimilar findings of the two regression analyses are not surprising in light of the relatively small overlap in variance of the two criteria of algebra achievement. Clearly, teacher-written algebra tests and standardized algebra achievement tests assess somewhat different domains of competence. The findings further suggest that some of the difference may be in the standardized tests' stronger emphasis on problem solving.

Some interesting sex-related differences emerged from the present investigation. Although the boys in the sample were more intelligent than the girls, as measured by an IQ test, and scored significantly higher on two of the three mathematics achievement tests in the seventh grade, the girls attained approximately equivalent grades in seventh-grade mathematics. Furthermore, the girls' algebra grades exceeded those of the boys despite the absence of a sex difference on the algebra achievement test. This relationship between sex and algebra grade remained significant after partialing out the effects of the cognitive predictors. Benbow and Stanley (1982) similarly found that girls in

their Study of Mathematically Precocious Youth achieved somewhat higher grades in mathematics courses even though the boys in their sample excelled on a mathematics reasoning test.

Contrary to reports that the academic performance of girls is generally more predictable than that of boys (Khan, 1973; Lavin, 1965), the product-moment correlations between predictors and criteria were somewhat larger (although generally not significantly) for the boys in the sample than for the girls. This finding is particularly surprising in view of a previous finding that the same prognosis test was a significantly better predictor of algebra grades for girls than for boys (Hanna & Lenke, 1970).

Finally, it is important to note that the multiple correlation coefficients yielded by the present study are somewhat lower than the .66 to .76 previously reported when grades were predicted (Barnes & Asher, 1962; Hanna et al., 1969; Mogull & Rosengarten, 1972; Sabers & Feldt, 1968) and .79 to .81 when achievement tests provided the criterion (Hanna et al., 1969; Sabers & Feldt, 1968). However, the samples in the earlier studies comprised ninth-grade students with a broader variation in academic abilities and aptitudes. The present sample of very talented eighth graders is a much more homogeneous group. No doubt then, the size of the zero-order and multiple correlations reflects the truncated range of values on the variables.

Such a modest level of predictability of eighth-grade algebra achievement also suggests that it will be necessary to explore alternative explanations for the failure of academically talented students in an eighth-grade algebra course. Perhaps algebra achievement is related to the ability to reason at the formal operational level described by Piaget (Inhelder & Piaget, 1958). Research has indicated that even in samples of above-average intelligence no more than half the eighth graders can be classified as formal operational (Flexer & Roberge, 1983; Roberge & Flexer, 1979). And a recent study of high-ability eighth graders has shown a relationship between level of operativity on a Piagetian test and mathematics problem-solving scores (Roberge & Flexer, 1983). Therefore, in light of the present study's finding that problem-solving ability is related to algebra achievement, the examination of the predictive power of a Piagetian test seems to be a promising direction for future research.

REFERENCES

Armstrong, J. M. (1981). Achievement and participation of women in mathematics: Results of two national surveys. *Journal for Research in Mathematics Education, 12,* 356–372.

Barnes, W. E., & Asher, J. W. (1962). Predicting students' success in first-year algebra. *Mathematics Teacher, 55,* 651–654.

Benbow, C. P., & Stanley, J. C. (1982). Consequences in high school and college of sex differences in mathematical reasoning ability: A longitudinal perspective. *American Educational Research Journal, 19,* 598–622.

Durost, W. N., Bixler, H. H., Wrightstone, J. W., Prescott, G. A., & Balow, I. H. (1970). *Metropolitan Achievement Tests.* New York: Harcourt Brace Jovanovich.

Flexer, B. K., & Roberge, J. J. (1983). A longitudinal investigation of field dependence-independence and the development of formal operational thought. *British Journal of Educational Psychology, 53,* 195–204.

Fox, L. H. (1975). Mathematically precocious: Male or female? In E. Fennema (Ed.), *Mathematics learning: What research says about sex differences* (pp. 1–12). Columbus, OH: ERIC Center for Science, Mathematics, and Environmental Education.

Hanna, G. S. (1972). *Modern Algebra Test.* Boston: Houghton Mifflin.

Hanna, G. S., Bligh, H. F., Lenke, J. M., & Orleans, J. B. (1969). Predicting algebra achievement with an algebra prognosis test, IQs, teacher predictions, and mathematics grades. *Educational and Psychological Measurement, 29,* 903–907.

Hanna, G. S., & Lenke, J. M. (1970, March). *Moderator variables in predicting algebra success.* Paper presented at the meeting of the American Educational Research Association, Minneapolis, MN.

Inhelder, B., & Piaget, J. (1958). *The growth of logical thinking from childhood to adolescence.* New York: Basic Books.

Khan, S. B. (1973). Sex differences in predictability of academic achievement. *Measurement and Evaluation in Guidance, 6,* 88–92.

Kovaly, M. J. (1980). Predicting Algebra I success using Grade 8 mathematics performance, teacher predictions, and a prognosis test. *Dissertation Abstracts International, 40,* 4939A. (University Microfilms No. 80-05, 468)

Lavin, D. E. (1965). *The prediction of academic performance.* New York: Russell Sage Foundation.

Lorge, I., Thorndike, R. L., & Hagen, E. (1964). *The Lorge-Thorndike Intelligence Tests.* Boston: Houghton Mifflin.

Mogull, R. G., & Rosengarten, W., Jr. (1972). Predicting student success in elementary algebra. *California Journal of Educational Research, 23,* 104–107.

Mogull, R. G., & Rosengarten, W., Jr. (1974). Predicting success in elementary algebra. *Alberta Journal of Educational Research, 20,* 34–38.

Orleans, J. B., & Hanna, G. S. (1968). *Orleans-Hanna Algebra Prognosis Test.* New York: Harcourt Brace Jovanovich.

Roberge, J. J., & Flexer, B. K. (1979). Further examination of formal operational reasoning abilities. *Child Development, 50,* 478–484.

Roberge, J. J., & Flexer, B. K. (1983). Cognitive style, operativity, and mathematics achievement. *Journal for Research in Mathematics Education, 14,* 344–353.

Rothenberger, M. L. (1967). Who should take algebra? *Clearing House, 42,* 226–228.

Sabers, D. L., & Feldt, L. S. (1968). The predictive validity of the Iowa Algebra Aptitude Test for achievement in modern mathematics and algebra. *Educational and Psychological Measurement, 28,* 901–907.

Swafford, J. O. (1980). Sex differences in first-year algebra. *Journal for Research in Mathematics Education, 11,* 335–346.

Taylor, C. L., Brown, F. G., & Michael, W. B. (1976). The validity of cognitive, affective, and demographic variables in the prediction of achievement in high school algebra and geometry: Implications for the definition of mathematical aptitude. *Educational and Psychological Measurement, 36,* 971–982.

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